Closing Wed: HW_3A,3B,3C (6.1-6.3)
Exam 1 is Thurs (4.9, 5.1-5.5, 6.1-6.3)

Entry Task:
Find the area of the region bounded by $4x = y^2$ and $y = 2x^3$ in 2 ways:
(i) Using dx
(ii) Using dy

$$4x = y^2 \iff \begin{cases} y = -2\sqrt{x} \\ y = 2\sqrt{x} \end{cases}$$

$$x = \left(\tfrac{1}{2}y\right)^{1/3} \iff y = 2x^3$$

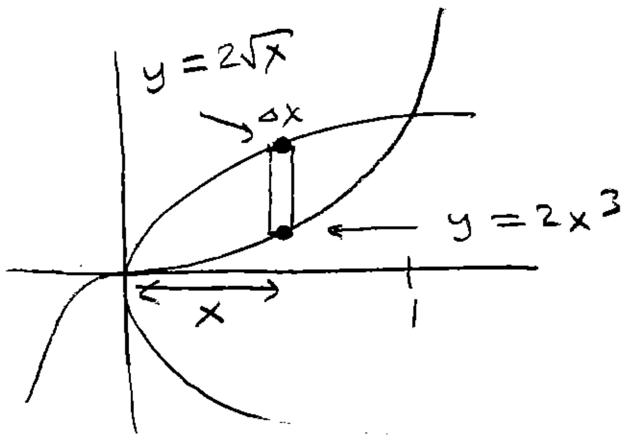
INTERSECTIONS $\Rightarrow$ $4x \overset{?}{=} (2x^3)^2$

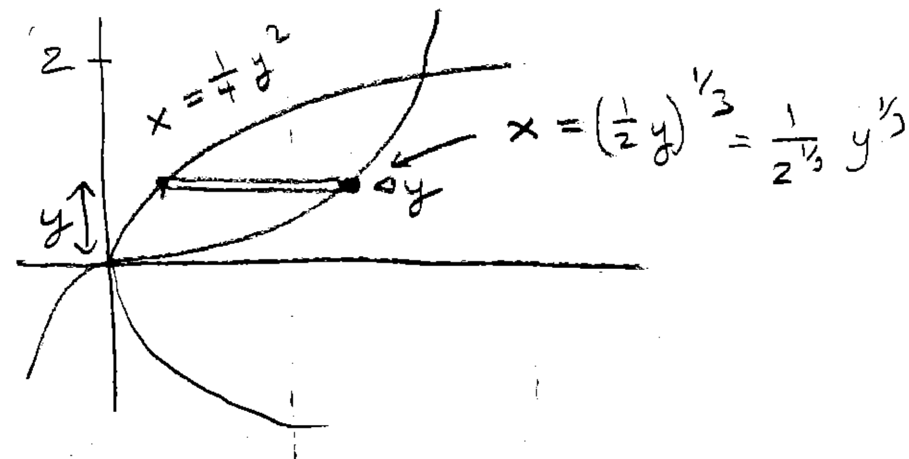
$4x = 4x^6$

$x = x^6$

$0 = x^6 - x = x(x^5 - 1)$

$x = 0$ or $x = 1$





$$\int_0^1 2\sqrt{x} - 2x^3\,dx$$

$$\frac{4}{3}x^{3/2} - \frac{2}{4}x^4 \Big|_0^1$$

$$4/3 - \frac{1}{2} = \frac{8}{6} - 3/6 = \boxed{5/6}$$

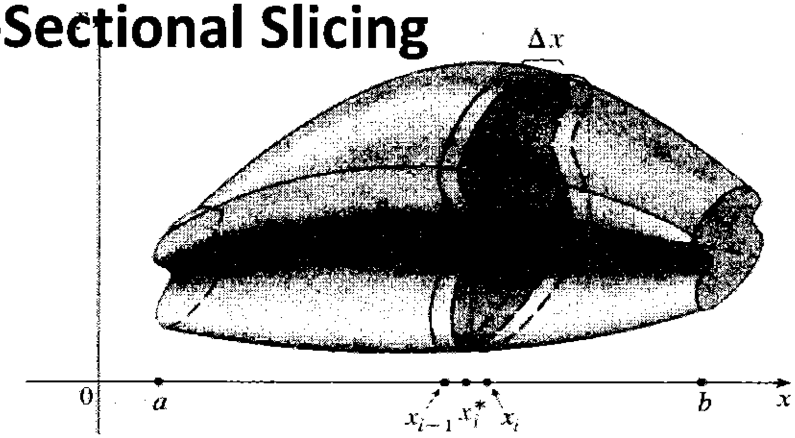
$$\int_0^2 \left(\frac{1}{2}\right)^{1/3} y^{1/3} - \frac{1}{4}y^2\,dy$$

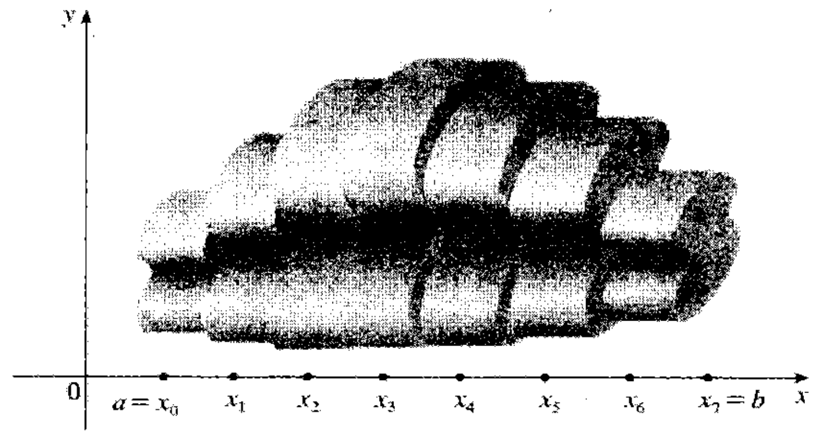
$$= \left(\frac{1}{2}\right)^{1/3}\frac{3}{4}y^{4/3} - \frac{1}{12}y^3 \Big|_0^2$$

$$= \left(\frac{1}{2}\right)^{1/3}\frac{3}{4}(2)^{4/3} - \frac{1}{12}(2)^3$$

$$= \frac{6}{4} - \frac{8}{12} = \frac{3}{2} - \frac{2}{3} = \frac{9}{6} - \frac{4}{6} = \boxed{5/6}$$

5.2 Finding Volumes Using Cross-Sectional Slicing





If we can find the general formula, $A(x_i)$, for the area of a cross-sectional slice, then we can approximate volume by:

$$\text{Volume of one slice } \approx A(x_i)\,\Delta x$$

$$\text{Total Volume } \approx \sum_{i=1}^{n} A(x_i)\,\Delta x$$

This approximation gets better and better with more subdivisions, so

$$\text{Exact Volume} = \lim_{n\to\infty} \sum_{i=1}^{n} A(x_i)\,\Delta x$$

We conclude

$$\text{Volume } = \int_a^b A(x)dx =$$

$$\int_a^b \text{"Cross-sectional area formula"}dx$$

Volume using cross-sectional slicing

1. Draw region. Cut **perpendicular** to rotation axis. Label *x* if that cut crosses the *x*-axis (and *y* if *y*-axis). Label **everything** in terms this variable.

2. Formula for cross-sectional area?

disc: Area = $\pi(\text{radius})^2$

washer: Area = $\pi(\text{outer})^2 - \pi(\text{inner})^2$

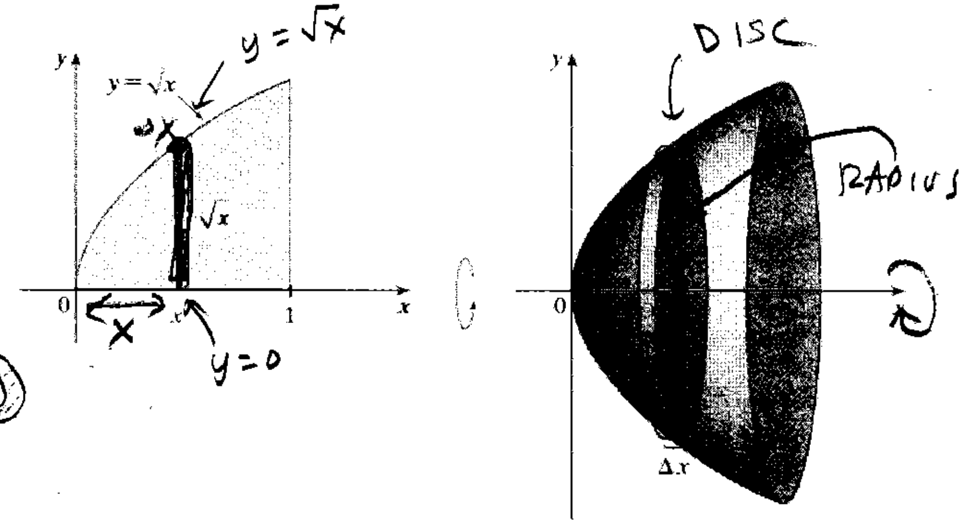
SAME

square: Area = (Height)(Length)

triangle: Area = ½ (Height)(Length)

3. Integrate the area formula.

Example: Consider the region, *R*, bounded by $y = \sqrt{x}$, y = 0, and x = 1. Find the volume of the solid obtained by rotating R about the **x-axis**.



(a) (b)

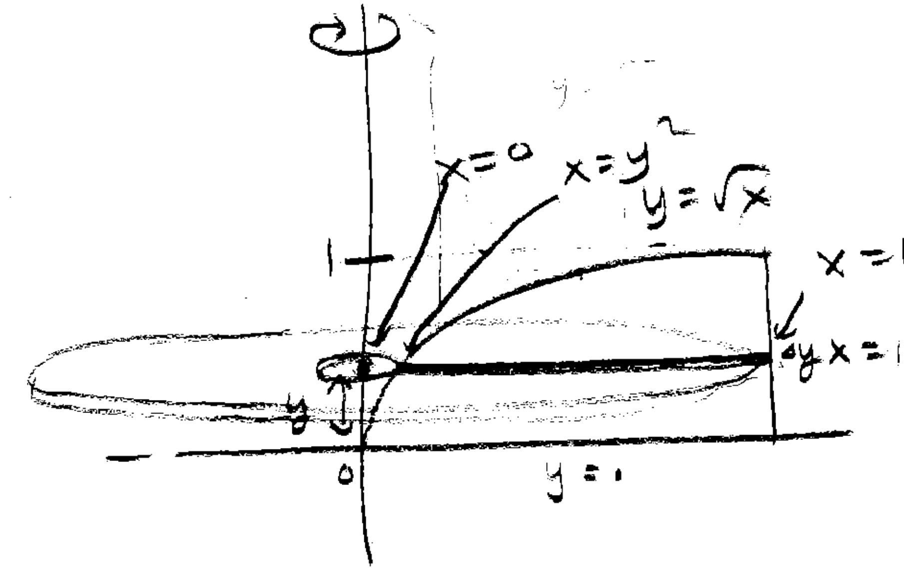
$$\int_0^1 \pi \, (\text{RADIUS})^2 \, dx$$

$$= \int_0^1 \pi \, (\sqrt{x})^2 \, dx$$

$$= \pi \int_0^1 x \, dx = \pi \, \tfrac{1}{2} x^2 \Big|_0^1$$

$$= \boxed{\pi/2}$$

Example: Consider the region, R, bounded by $y = \sqrt{x}$, y = 0, and x = 1. Find the volume of the solid obtained by rotating R about the **y-axis**.

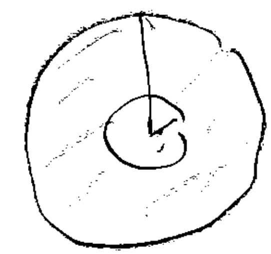


$$\int_0^1 \pi(1)^2 - \pi(y^2)^2\,dy$$

$$= \pi \int_0^1 1 - y^4\,dy$$

$$= \pi\left(y - \tfrac{1}{5}y^5\Big|_0^1\right)$$

$$= \pi\left(1 - 1/5\right) = \boxed{4\pi/5}$$



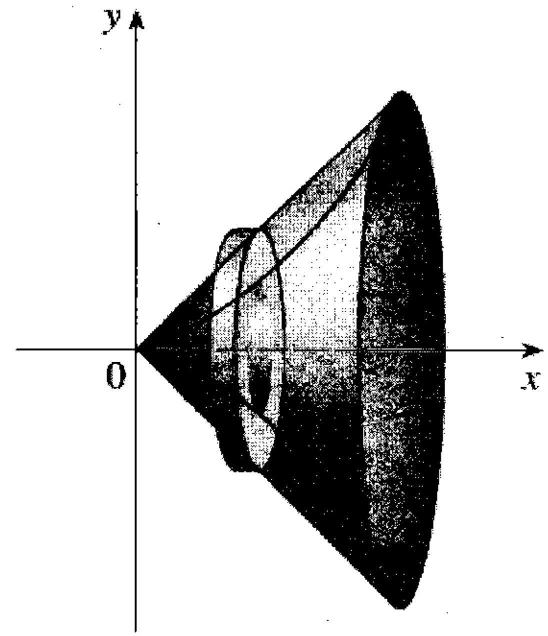
Example: Consider the region, R, bounded by $y = x$ and $y = x^4$. Find the volume of the solid obtained by rotating R about the **x-axis**.

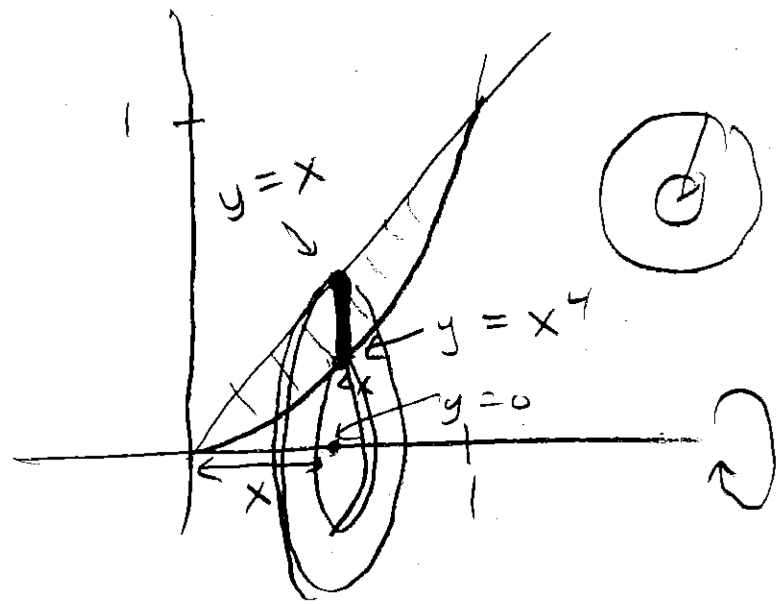
$$\int_0^1 \pi(x)^2 - \pi(x^4)^2 \, dx$$

$$\pi \int_0^1 x^2 - x^8 \, dx$$

$$\pi \left(\frac{1}{3}x^3 - \frac{1}{9}x^9 \right) \Big|_0^1$$

$$\pi \left(\frac{1}{3} - \frac{1}{9} \right) = \boxed{\frac{2\pi}{9}}$$



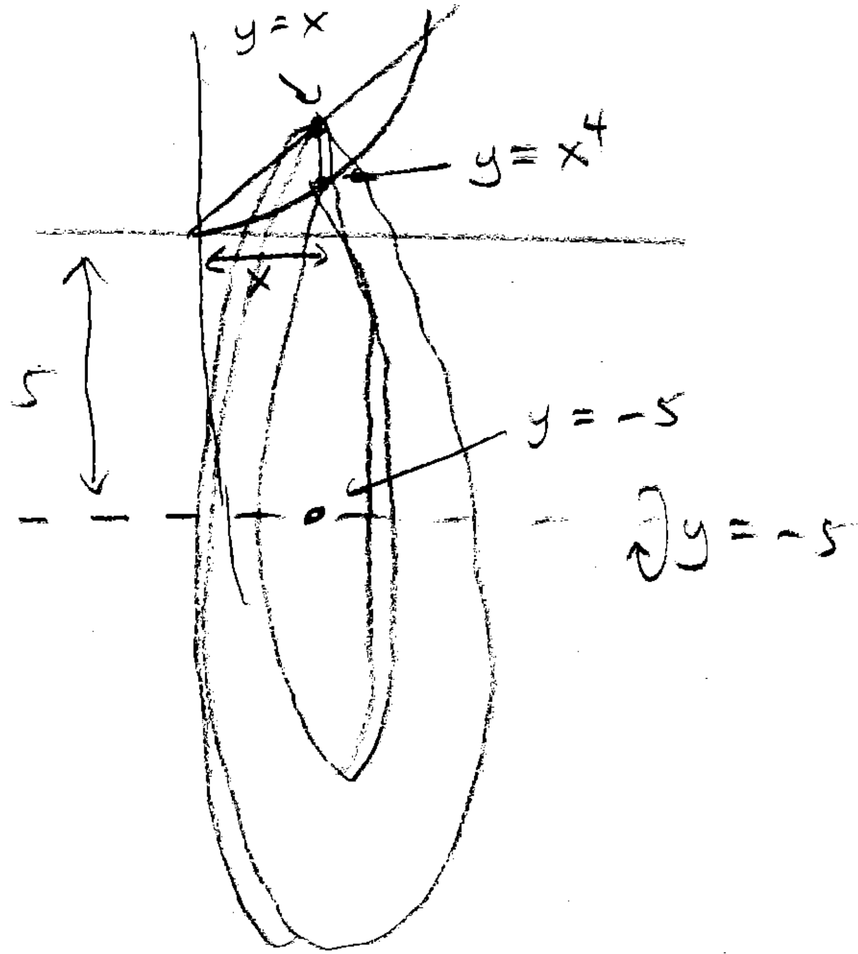


Example: Consider the region, R, bounded by $y = x$ and $y = x^4$. (R is the same as the last example).

(a) Now rotate about the horizontal line y = -5. What changes?

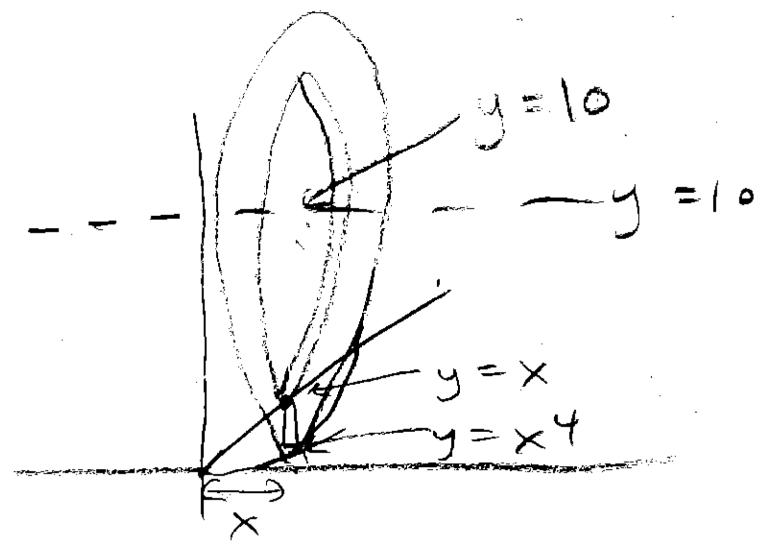
$$\int_0^1 \pi(x - -5)^2 - \pi(x^4 - -5)^2\,dx$$

$$\pi \int_0^1 (x+5)^2 - (x^4+5)^2\,dx$$



(b) Now rotate about the horizontal line y = 10. What changes?

$$\int_0^1 \pi(10 - x^4)^2 - \pi(10 - x)^2\,dx$$

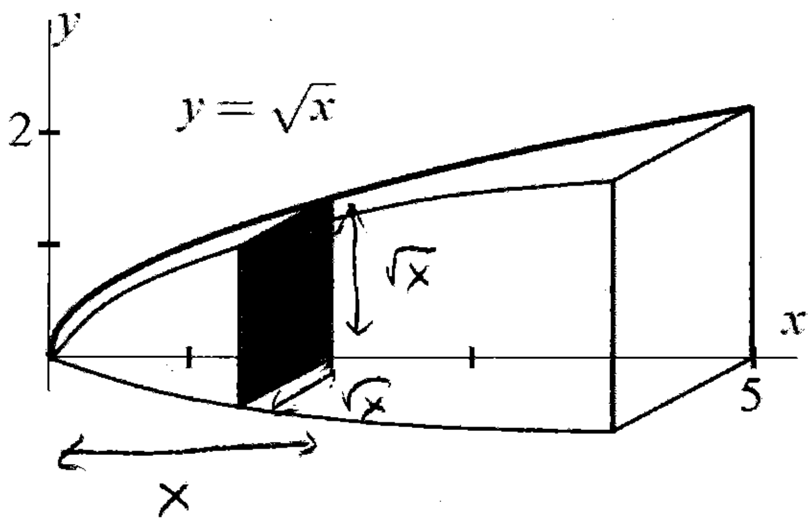


Example:

(From an old final and homework)

Find the volume of the solid shown.

The cross-sections are squares.



$$\int_0^5 (\text{HEIGHT})(\text{LENGTH})\, dx$$

$$\int_0^5 \sqrt{x}\ \sqrt{x}\ dx$$

$$\int_0^5 x\, dx = \frac{1}{2}x^2\Big|_0^5 = \frac{1}{2}25 = \boxed{\frac{25}{2}}$$

Summary (Cross-sectional slicing):

1. Draw Label
2. Cross-sectional area?
3. Integrate area.

This method has a major limitation:

6.2 method about *x-axis*, must use *dx*.

6.2 method about *y-axis*, must use *dy*.

What if the regions is rotated about the *x*-axis and we need to use *dy*? (or about *y*-axis and we need dx?)

In these cases, 6.2 "Cross-sectional slicing" wouldn't work!

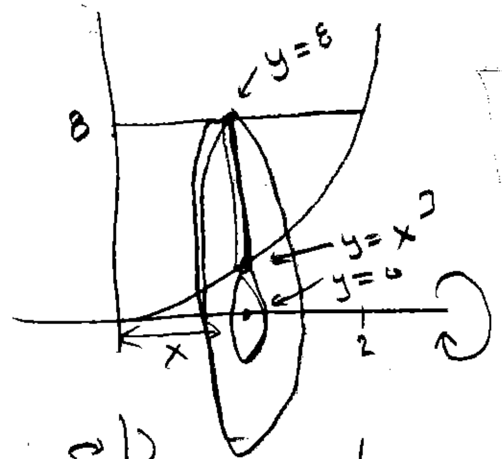
We need another method.

That is what we will do in 6.3.

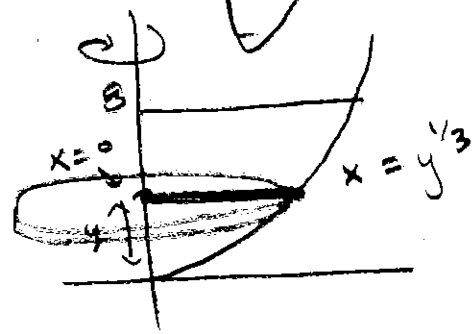
Close Wed: HW_3A,3B,3C
(complete sooner!)
Exam 1 is Thurs (4.9, 5.1-5.5, 6.1-6.3)

Entry Task:
Consider the region *R* bounded by
$y = x^3, y = 8$, and $x = 0$.
Set up the integrals that would give the volume of the solid obtained by rotating *R* about the ….

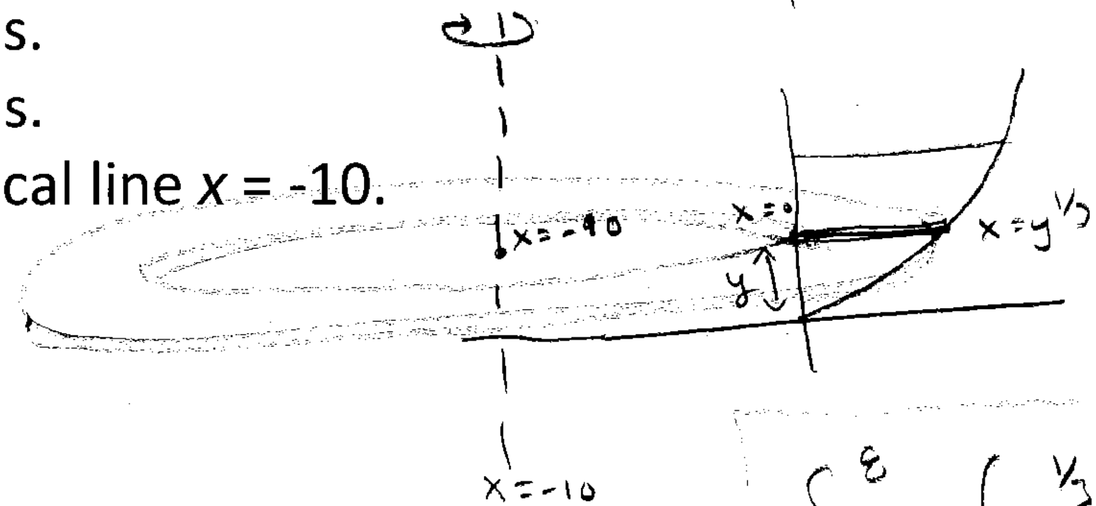
(a) … *x*-axis.
(b) … *y*-axis.
(c) … vertical line *x* = -10.



$$\boxed{\int_0^2 \pi(8)^2 - \pi(x^3)^2\,dx \qquad \pi\int_0^2 64 - x^6\,dx}$$



$$\boxed{\int_0^8 \pi\left(y^{1/3}\right)^2 dy = \pi\int_0^8 y^{2/3}\,dy}$$



$$\boxed{\int_0^8 \pi\left(y^{1/3}+10\right)^2 - \pi(10)^2\,dy}$$